A mathematical model for the outflow of aqueous humor from the anterior chamber to Schlemm’s canal

Sapna Ratan Shah, Shabab Akbar
Biomathematical Lab No. 34, School of Computational and Integrative Sciences, Jawaharlal Nehru University, New Delhi, INDIA

Abstract: A study of aqueous humor outflow passing through the trabecular meshwork in the canal of Schlemm’s is presented in this paper. Canal of Schlemm’s is connected by the trabecular meshwork and that passage is considered as a permeable compliant channel. The internal wall of the canal is porous as well as aqueous and humor percolates along with it. The aqueous humor is filtering through the canal segment before attaining to the collector channel. This present model suggested flow in a canal section in the middle of two different collector channels. The first one is the Elliptic passage collector channel and the second is the annular passage collector channel. Aqueous humor fluid pressure as well as volume flux and the effects of filtration constant, intraocular pressure is discussed in this present paper. Due to the increased intraocular pressure, the more aqueous humor has to percolate into the canal via the permeable innermost layer and this extra percolation affects the volume flux of aqueous humor in the canal of Schlemm. It is observed in this work that aqueous flux rises with the increased percolation of aqueous humor inside the Schlemm’s canal.

Keywords: Schlemm's canal, Trabecular meshwork, Aqueous humor flow, Collector channel, Aqueous fluid pressure.

I INTRODUCTION

Primary Open Angle Glaucoma (POAG) give rise to blindness and continuously affecting 70–75 million patients globally. In order to understand, Primary Open Angle Glaucoma (POAG), it is important to understand the anatomy of eye, formation, and drainage of aqueous humor (AH) in the eye [Fig. 1]. The eyes are organs of the visual system and aqueous humor is an optically clear, slightly alkaline ocular thin fluid like water, it is similar to the plasma of the blood and located between the anterior chambers and posterior chamber of the eye. Aqueous humor flows through the ciliary fibers, towards the posterior chamber in the middle of the lens and the iris. From this chamber, the aqueous humor fluid flows out of the pupil and get into the anterior chamber [1,3]. The ciliary body builds around 2.5 μL of aqueous humor each minute and the ciliary body continuously produces aqueous humor in the eye by the ciliary processes. It is a transparent substance continuously produced by the ciliary epithelium approximately 2.3 μlitre/min. It flows behind the iris in the posterior chamber and drains from the eye through the drainage angle. Aqueous humor starts flowing from that angle and passes by way of biological filter which is known as trabecular meshwork (TM) toward the Schlemm’s of Canal (SC), which is the main drainage passage from the eye and lastly reaches to the “collector channels” [2,6]. Regular production and drainage occurrence of this aqueous humor fluid in the eye is an essential process to ensure the stability of the IOP which is very necessary to support the visual activity of the eye and to provide the nutrition to the tissues to the eye. The average intraocular pressure of a healthy human is about 12-22 mmHg. In a healthy eye, the rate of secretion of aqueous humor balances the rate of drainage of aqueous humor. When this drainage process somehow shuts off the aqueous humor cannot go out quickly hence the intraocular pressure (IOP) rises in the eye [8,11].

Fig. (1). Human eye

The excessive production of aqueous fluid in the anterior chamber of the eye raises the intraocular pressure (IOP). If this raises intraocular pressure retained for a very long period then it leads to destruct the optic nerve and may cause the Primary Open Angle Glaucoma, which is the main reason for the loss of sight. The patients with Glaucoma, the drainage of aqueous humor from the trabecular meshwork is partially or entirely choked. Aqueous humor fluid increases in the chambers and due to this, the pressure increases inside the eye. This pressure pushes the lens system in the posterior body and the lens pushes the vitreous body which compresses the eye and as a result, the blood vessels of the eye and nerve fibers are disturbed. These disturbed and damaged blood vessels and nerve fibers result in Glaucoma. Primarily, the glaucoma is of two types. (1) Open-angle glaucoma and (2) Closed-
angle glaucoma [4]. Open-angle glaucoma retains lifelong in the eye. It is initiated by partially blocking the drainage of the canal. The angle between the iris and the cornea is wide open which means the entrance at the drainage is unblocked, but the outflow of aqueous humor fluid is slightly sluggish. The pressure increases progressively in the eye for too long period. Symptoms start appearing gradually from peripheral vision loss and can go on until the loss of central vision [5,9]. Growth of glaucoma can be pause with medical treatments, but part of the image that has already been lost cannot get back. This is why it is crucial to detect the signs of glaucoma early with regular eye exams. The closed-angle glaucoma is inspired by a sudden complete blockage of AH drainage. The pressure in the eye rises rapidly and leads to vision loss rapidly. This happens when the pupil dilated and stuck to the back of the iris. This prevents the AH from flow into the Anterior Chamber. Accumulation of the aqueous humor into the posterior chamber presses on the iris results in the block the angle fully. This Closed Angle Glaucoma is a medical emergency and required instant attention. In both cases, intraocular pressure (IOP) raises. In the case of Open-Angle Glaucoma, the most effective technique to lower down the IOP is by attacking the inflow system with drugs that decreases the rate of production and secretion of aqueous humor.

II. PROBLEM FORMULATION AND SOLUTION

Nearly all of the aqueous humor fluid running through the canal of Schlemm has to flow some extent along the canal to get into the collector channel. There are two kinds of channel represented in this model, circular channel and an elliptical channel between the two collectors of a canal [Fig. 2, 4]. Almost half of the total amount of the humor percolates through the canal and reaches the collector to the right of the midpoint between two collectors and another half to the left side. Hence the flow at the midpoint, described by \( z = 0 \), is zero [Fig. 3]. Thus, aqueous humor flow in the section is regarded as fluid flow through a narrow shape of elliptic and annular passage. The following assumptions have taken in the formulation of the problem:

- The flow is taken into consideration in half of the segment in \( z \)-direction due to symmetry.
- The damage in the inner layer of the canal of Schlemm is proportional to the pressure drop.
- The aqueous fluid flow is Newtonian, laminar, steady, incompressible, and viscous.
- The size of the collector channels is the same.
- All collector channels are draining the same amount of the aqueous humor fluid.
- The innermost endothelium layer of the canal is permeable, collapsible, and porous.
- All the collector channels are onf equal distance.

The governing equations of fluid mechanics (continuity and motion):

The governing Navier-Stokes equation which is a partial differential equation stating the local balance of the momentum in the fluid around any time any point in the space is given by:

\[
\frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \frac{\mu}{\rho} \nabla^2 \mathbf{v} + \mathbf{f}
\]  

(1)
Where, \( \frac{Dv}{Dt} \) = Instantenous
\(-7p = Dynamic Pressure\)
\( \mu \frac{\partial v}{\partial t} = Shearviscosityterm\)
\( f = External force\)
\( \rho = Density\)

By introducing the assumptions in the equation (1), the leading Navier–Stokes equations represented as:
(2)
(3)
(4)
The equation of continuity is reducing as:
(5)
From the material balance equation:
(6)
where \( \alpha \) denotes the filtration flux of the aqueous humor and \( \beta \) and denotes the aqueous humor volume flux in the canal.

By expansion of \( q(z + dz) \) in a Taylor series, we have:
(7)
where,
\[
q(z) = \int v_z \, dx \, dy,
\]
The boundary conditions are given below:
(8)
where \( P_0 \) is the pressure at the entrance.

The case I: Elliptic passage: Equation no. (1) and (2) represent that the pressure, the governing differential equation is reduced to the form:
(9)
For the elliptic boundary \( v_z \) is
(10)
Using Equation no. (12) in Equation. (11),
(11)
By using the value of \( k \) in equation (10),
(12)
And
(13)
Aqueous outflux is:
(14)
The elliptical boundary, from Eq. (6)
(15)
From boundary conditions in equation no. (8) and (13), the differential equation is as below:
(16)
where, \( m = \left( \frac{2\mu(\alpha + b)(\alpha^2 + b^2)}{a^3 b^3} \right)^{1/2} \)
The solution of Eq. (16) with the boundary conditions is given as:
(17)
And also
(18)
(19)
The case II: Annular passage: In this case, it is assuming \( a = b \)
and now expression for volume flux and pressure are:
(20)
where \( m = \left( \frac{2\mu}{a^3 b^3} \right)^{1/2} \)
and \( r \) is the canal’s radius.
To find the solution of Eq. (17), we have,
(21)
(22)
(23)
III. RESULTS AND DISCUSSION
The model has shown before contributes to the fact that the increased aqueous flux and pressure contribute to damaging the optic nerve that results in Glaucoma. Therefore, the incorporation of elliptic passage cross-sectional tube and annular passage cross-sectional tube describe the simplest representation of aqueous humor flow in the eye from the canal of Schlemm's segment to collator channel through the trabecular meshwork. The effects of significant parameters like intraocular pressure of the normal eye, filtration constant on pressure profile, and volume flux profile in the elliptic and circular passage canal have been investigated and discussed. The analytical results obtained in this study consist of the expression for volume flux and pressure in two different cases for elliptical and circular cross-sectional tube in equation no. (18), (19), (22), (23) and display through graphs from Fig. (5) to Fig. (12).

The two different cases are:
(i) The porous inner layer of the canal passage is collapsible and change into the shape of an elliptic passage tube.
(ii) The porous inner layer of canal passage is rigid and change into the shape of an annular passage tube.
The different values of parameters used in this present paper are shown in Table 1.

Fig. (5) to Fig. (8) consist of the variation of pressure profile, volume flux with axial distance for different parameters of filtration constant, and intraocular pressure in the elliptical channel. And the variation of pressure profile, volume flux with axial distance for different parameters of filtration constant, and intraocular pressure in the circular channel have shown by figures from Fig. (9) to Fig. (12).
It is evident from Fig. (5), Fig. (6), Fig (9), and Fig. (10) that the pressure increases and volume flux increases as distance increases [12]. It should also be noted here that the pressure and volume flux increase as filtration constant and intraocular pressure increases. The results are therefore consistent with the observation of [10]. Fig. (7), Fig. (8), Fig. (11) and Fig. (12) describe that the volume flux and pressure increase as the filtration constant and intraocular pressure increases. It is also noticed here that the pressure and volume flux increase as distance increases and these results are compared with the [7]. The aqueous humor volume flux rises in Schlemm's canal with an increase of intraocular pressure. The filtration constant rises, as well as the aqueous percolation along with the inner layer of the wall rises [10]. And due to this extended percolation, the aqueous flux raises in the canal and it is demonstrated by the curves in Fig. (7) and Fig. (8). The volume flux, as is detected from the graph in Fig. (11) and Fig. (12) that when (IOP) will increase or arise, the filtration constant (G) forces enough
Fig. (9): Pressure profile with axial distance for different values of filtration constant in a circular channel.

Fig. (10): Pressure profile with axial distance for different values of intraocular pressure in a circular channel.

Table 1: Value of parameter and description for aqueous humor flow

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aqueous humor pressure in collector channel ($P_0$)</td>
<td>12 mmHg</td>
</tr>
<tr>
<td>Filtration constant (G)</td>
<td>8.28182 mm² s/g</td>
</tr>
<tr>
<td>Distance between two collector channels (2l)</td>
<td>1200 μm</td>
</tr>
<tr>
<td>Dynamic viscosity of aqueous humor ($\mu$)</td>
<td>0.75 cp</td>
</tr>
<tr>
<td>Value of parameter for major axis of the elliptic passage (a)</td>
<td>0.15 mm</td>
</tr>
<tr>
<td>Value of parameter for the minor axis of the elliptic passage (b)</td>
<td>0.005 mm</td>
</tr>
<tr>
<td>Value of parameter for the radius of the annular passage (r)</td>
<td>0.13 mm</td>
</tr>
</tbody>
</table>

Fig. (11): Volume flux with axial distance for different values of filtration constant in a circular channel.

Fig. (12): Volume flux profile with axial distance for different values of intraocular pressure in circular channel.

Due to the increased intraocular pressure, the optic nerve is damaged and results in Glaucoma. In this case, the more aqueous humor has to percolate through the canal passage into the porous inner layer and this developed percolation affects the aqueous volume flux of the canal. The aqueous flux rises with increased percolation of the aqueous humor in the canal. The pressure profile and the volume flux are

IV. CONCLUSION

Due to the increased intraocular pressure, the optic nerve is damaged and results in Glaucoma. In this case, the more aqueous humor has to percolate through the canal passage into the porous inner layer and this developed percolation affects the aqueous volume flux of the canal. The aqueous flux rises with increased percolation of the aqueous humor in the canal. The pressure profile and the volume flux are
more for the elliptic passage channel than the annular passage channel.

References: