The Producing functionals in solutions of some extreme problems connected with heat conduction and Navier-Stokes equations and some others processes

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Abstract—The work are devoted to solutions equations of heat conduction and Navier-Stokes by Producing functionals method. Besides are considered by questions modeling a tree of numbers arising at the analysis of the numerical and corresponding text information. It is shown that many questions of mathematical analysis problems and their applications are reduced to construction of Producing functionals models in extreme regimes and connected with it construction of Model Numbers Tree. Corresponding formulas are received for equations of heat conduction and Navier-Stokes in the polynomial form. Given others applications from physical, hydromechanical, labor resource and others processes.

Index Terms—class so-called producing functionals, transformations, models equations, heat conduction, Navier-Stokes, polynomial form, hydromechanical and labor resource processes.

Let is given set $[1-3]$ $\mathcal{M} = \left \{ (a_1, \ldots, a_m) : m \sum_{j=1}^{m} a_j x_j^{n_j} = 1, \alpha_j(t) \geq 10, j = 1, m \right \}$

$m, n, s$ are natural, $t \in T, T$ is a arbitrary set from $[0, \infty)$. Let $\alpha \in M = M^s$ and $\alpha, x \in L_m^n(T)$ with norm $\|x\| = \left( \int_T \sum_{j=1}^{m} |x_j|^n dt \right)^{1/n} \leq \infty$. Assume that defined the set of producing functionals of type

$$\mu(\alpha) = \left( \int_T \sum_{j=1}^{m} \alpha_j \frac{|x_j|^n}{\|x\|^n} dt \right)^{1/2},$$

are given for all $\alpha \in M^s$ and $x \in L_m^n(T)$. The set of functional (1) with norm $\|\mu\| = \sup_{\alpha \in M^s} \mu(\alpha)$ is the normed space. It denote by $\mathcal{M}$. Introducing the vector $y = K|x|$, where $K$ is the diagonal matrix with elements $\alpha_j^{1/n}$ we have the space with norm $\mu(\alpha) = \|y\|_{L_m^n}(T) = \left( \int_T \sum_{j=1}^{m} |y_j|^n dt \right)^{1/n}$ is also the normed space. It denote by $\mathcal{M}^s(\alpha)$.

It is known that $\alpha = \alpha^0 \in M^s$, where $\alpha^0 = \left( \sum_{j=1}^{m} \frac{|x_j|^n}{\|x\|^n} \right)^{2/n}, j = 1, m$ the functional (1) has a maximal value $\|\mu\| = \left( \int_T \sum_{j=1}^{m} |x_j|^n dt \right)^{1/n} = \|x\|_{L_m^n}(T), \|y\|_{L_m^n}(T) \leq Z, \|y\|_{L_m^n}(T) \leq Z$ and what is more all maximal values of the functional (1) with different $x \in L_m^n(T)$ are solution of the equation $\sum_{j=1}^{m} X_j^n = Z^n$ where $X_j = \left( \int_T |x_j|^n dt \right)^{1/n}, j = 1, \ldots, m; Z = \|\mu\|$. Besides $M \in M^s(\alpha)$ for all $\alpha \in M^s$ (1).

Introducing $X_j = \left( \int_T |x_j|^n dt \right)^{1/n}, j = 1, m, Z = \|\mu\|$ we have the equation:

$$\sum_{j=1}^{m} X_j^n = Z^n,$$

It is to easy that $M \in M^s(\alpha)$ for all $\alpha \in A$. Now we consider the model space of functionals $\mathcal{M}$ with norm

$$\mu(\alpha) = \left( \int_T \sum_{j=1}^{m} |x_j|^n dt \right)^{1/2},$$

where $x \in L_m^n(T), \beta_j(t) = \int_0^t \alpha_j(t) dt, \alpha \in A = A^s$. The set of points $x_1, \ldots, x_m$ with norm $\beta$ is formed the Euclidean model space $M^{1,n}(\alpha), \alpha \in A$. For functionals of type (3) from $\mathcal{M}$ also take place $\mathcal{M} = M^s(\alpha)$.

For any natural $n > 1$ between to set of solutions (4) ( or (2) ) ( i.e. under $m = k - 1$ and $m = k$ ) it take place next presentations:

$$\hat{x}_{jk} = \hat{x}_{12}\hat{x}_{jk-1}, \hat{x}_{kk} = \hat{x}_{22}\hat{x}_{k-1},$$

$$\hat{s}_k = \hat{z}_2\hat{s}_{k-1}, j = 1, k - 1$$

where $k = 3, 4, \ldots, m$, $(\hat{x}_{12}, \hat{x}_{22})$ is some point of the special Plane with distance $\hat{z}_2 = (\hat{x}_{12}^2 + \hat{x}_{22}^2)^{1/2}$. Let $(x_{1k-1}, \ldots, x_{k-1,k-1}, x_{k-1})$ are solutions (4) under $m = k - 1$ ( the sign of $\hat{z}$ is dropped ). We shall that $(x_{ik}, \ldots, x_{kk}, x_k)$ obtained by of (5) and are the solutions (4) under $m = k$. So that $\sum_{i=1}^{k-1} x_{ik-1} = \hat{z}_{k-1,1}$, then multiplied both part of the last identify on $x_{1k}^2$ we have: $x_{12}^2 \sum_{i=1}^{k-1} x_{ik-1}^2 = \int_T \frac{1}{\|\mu\|^n} |x|^n dt$.
In the process of transformation, it is necessary to ensure that the initial and boundary conditions are satisfied. The solution of the transformed equation is then used to determine the solution of the original equation. This method is particularly useful when the original equation is difficult to solve directly.
The disorder of energy and pressure will be maximal.
Let function \( u = u(x, t) \), \( t \geq 0, \ x = (x_1, x_2, \ldots, x_m) \), \( x \in G, G \subseteq E^m \) is a state of some object (or process) in a point \( x \) at the moment of time \( t \). Now we consider Navier-Stokes equation
\[
\frac{\partial u_i}{\partial t} + \sum_{j=0}^{3} u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \sum_{j=0}^{3} \frac{\partial^2 u_i}{\partial x_j^2} - \frac{\partial u_i}{\partial t}, \quad -\infty < x_i < \infty, t > 0.
\]
Let \( u = \sum_{i=0}^{3} \alpha_i u_i, \alpha \in M, \) where \( M = \{ \alpha : \alpha = (\alpha_1, \ldots, \alpha_m), \sum_{i=0}^{m} \alpha_i^2 = 1 \} \). Then we have \( \frac{\partial u}{\partial t} + \sum_{j=0}^{3} u_j \frac{\partial u}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \sum_{j=0}^{3} \frac{\partial^2 u}{\partial x_j^2} \). We also suppose that \( u_i = \delta_i u \), where \( \delta_i \in M \). Then \( \sum_{j=0}^{3} \frac{\partial^2 u}{\partial x_j^2} - \frac{\partial u}{\partial t} \). We shall minimize left part of last equation on parameters \( \delta \in M \), and we have
\[
\sum_{j=0}^{3} \left( \frac{\partial^2 u}{\partial x_j^2} \right)^n = \left( \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \sum_{j=0}^{3} \frac{\partial^2 u}{\partial x_j^2} - \frac{\partial u}{\partial t} \right)^n
\]
For solution of this equation we consider next class possibility solution: 1. \( \frac{\partial^2 u}{\partial x_j^2} = c_j \) the simple class and 2. \( \frac{\partial^2 u}{\partial x_j^2} = c_j u^2 \) the exponential class, where \( \sum_{j=0}^{3} c_j^2 = c^2 \) is coordination equations - algebraic representation.
In the beginning we shall consider the exponential class
\[
\frac{\partial^2 u}{\partial x_j^2} = c_j u^2, \quad \frac{\partial u}{\partial t} = \nu \sum_{j=0}^{3} \frac{\partial^2 u}{\partial x_j^2} + \frac{1}{\rho} \frac{\partial P}{\partial x}
\]
Such we can take \( \sum_{j=0}^{3} \alpha_j \frac{\partial P}{\partial x_j} + \nu \sum_{j=0}^{3} \frac{\partial^2 u}{\partial x_j^2} - \frac{\partial u}{\partial t} \). We have
\[
\sum_{j=0}^{3} \left( \frac{\partial P}{\partial x_j} \right)^n = \left( \rho \sum_{j=0}^{3} \frac{\partial^2 u}{\partial x_j^2} - c u \right)^n.
\]
It allows proving, that the decision exists and is enough smooth function and it will allow essentially changing ways of realization hydroaerodynamic calculations. We shall write some classes of possible solutions (them much - by virtue of a generality of the description of real processes by this the equation) for the initial equation: \( \frac{\partial^2 u}{\partial x_j^2} = c_j \), the simple class, \( \frac{\partial^2 u}{\partial x_j^2} = c_j u^2 \), the exponential class \( \frac{\partial P}{\partial x_j} = d_j \), the simple class \( \frac{\partial P}{\partial x_j} = d_j u \), \( \frac{\partial P}{\partial x_j} = d_j P \), the exponential class, and \( \nu \sum_{j=0}^{3} \frac{\partial u}{\partial x_j} - \frac{\partial u}{\partial t} = c \), the simple class, \( \nu \sum_{j=0}^{3} \frac{\partial u}{\partial x_j} - \frac{\partial u}{\partial t} = c u \), the exponential class, \( \nu \sum_{j=0}^{3} \frac{\partial u}{\partial x_j} - \frac{\partial u}{\partial t} = c P \), the mixed class where \( c_j, d_j, j \) be show solutions the so-called equation of the coordination of capacity (the coordination equation):
\[
\sum_{j=1}^{n} c_j^n = c^n
\]
and \( \sum_{j=1}^{n} d_j^n = d^n \) and for a simple class we shall write the appropriate general and smooth solution: \( u(x, t) = \sqrt{\left( \int_{-\infty}^{\infty} G(x, \xi)u(x, \xi)\,d\xi + \int_{0}^{t} dt \int_{-\infty}^{\infty} G(x, \xi)(c + d + d\xi)\,d\xi \right)^2 + Q} \),
\[
Q = \sum_{j=1}^{n} c_j x_j, \quad P(x) = P(0) + \sum_{j=1}^{n} \alpha_j u_j, \quad G(x, \xi) = \left( \frac{1}{2\sqrt{\pi \rho}} \right)^n e^{\frac{1}{4}(\sum_{j=1}^{n}(x_j - \xi))^2}
\]
is the sources function.

b) The equation of Navier-Stokes. The considering equation belongs to as class difficult equations and is the basic at calculation of movement viscous incompressible liquids consisting of three equations which are written down as one vector equation. However generally it is not solved methods of modern mathematics and in practice it is necessary to be limited to the decision of only private problems. Solutions of these equations are unknown, and thus even it is not known, how them to solve. However we have found classes of possible general and smooth solutions that equation on the basis of entered earlier to us a principle of extreme conditions that takes place in processes and objects of the real world. We shall propose that:

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physics and even in economy. This equation is defined from continuity equation. Really, under integration of this equation we have: \( w(x,t)u(x,t) = w(0,t-x/u)h(0,t-x/u) \). The condition is given for determination of this function. Now we consider the discrete equation of with the begin condition for some function \( \varphi(x) \). Next representation takes place: 

\[
\begin{align*}
Q(x,t) &= C_0 e^{\delta_{\max} (t-x/u)} + \sum_{j=1}^{\infty} C_j e^{\alpha_j (t-x/u)} \cos \beta_j (t-x/u),
\end{align*}
\]

where \( \delta_{\max}, \delta_j = \alpha_j + i \beta_j \) are the roots of the equation \( \int_{-x}^{x} \varphi(x) e^{-\delta x/u} dx = 1, \) more over \( |\alpha_j| \leq \delta_{\max} \), \( C_j \) are Fourier’s coefficients of the function \( Q(x,0), Q(0,0) = Q_0, j = 1, 2, \ldots \). It is noticed that formula and these equation are received. Now we shall solved the equation for some distributions laws. From the beginning we consider the case when 

\[
\varphi(x) = \begin{cases} 
\frac{1}{2\pi} & \text{at } x \in [-x_s, x_s], \\
0 & \text{at } x \notin [-x_s, x_s],
\end{cases}
\]

and for definition roots we have \( sh(\alpha_j + i \beta_j)x_s = 1. \) Hence

\[
\alpha_j = \frac{\arcsinh 1}{x_s}, \quad \beta_j = \frac{2\pi j}{x_s}, \quad j = 0, 1, 2, 3, \ldots
\]

Now we consider the case when 

\[
\varphi(x) = \frac{1}{\sqrt{2\pi c}} e^{-\frac{x^2}{2c}}
\]

As \( \delta_{\max} \varphi(x) = \frac{1}{\sqrt{2\pi c}} e^{-\frac{x^2}{2c}} \), then \( \sigma = x_s/3 \) and for estimation of the quantity of the perturbed water we take the first wave:

\[
Q(x,t) = Q_0 e^{\delta_{\max} (t-x/u)}
\]

Under supposition \( h(x,t) = h(0,t-x/u) \) from movement equation we have next formula:

\[
u(x,t) = \sqrt{(c_0^2 H - 2g)h},
\]

where \( c_0 \) is Shezi coefficient, \( h \) is power of unperturbed water, \( H \) is power of perturbed wave. Note that \( (c_0^2 H - 2g)h \geq 0 \) under \( H \geq H_0 = 4mm - 5mm \) and \( u \geq 0 \). Under \( H = 4mm - 5mm \) we have \( u = 0 \). Besides \( u = c_1 Q^{1/5}, h = c_2 Q^{2/5}, \) \( c = c_3 u \), where \( c_i = 1, 2, 3 \) are constants, \( c \) is the speed of perturbed wave, \( c_3 = 49 \) is noticed that our results are used under mathematical modeling of Sarez Lake on the Pamir mountains. It is known that on the February 18, 1911, the territory of Pamir was struck by an earthquake with a magnitude of 9.9 on the MSK scale (t=18 ours 41 minutes 14 seconds; \( \phi = 38^\circ N; \lambda = 72.8^\circ E; H = 26km, M = 7.4; I = 9 \in \) In the region of the village Usoi, the earthquake triggered a grandiose landslide (over 2 billion \( m^3 \) of rock) that bloiced the valley of the river Murgab and buried the village Usoi with all its inhabitants in the deis avalanche. The barrage built by the rockslide masses was named Usoi obstruction after the biured village The water of the river Murgab began to accumulate behind the barrage, and flooded one of the largest settlements of the valley, village Sarez. Thus it was formed Lake Sarez with next parameters[1,2]: High water mark=3.265 m, water catchment area = 16.506 km², lake surface =80 km², length =602 km, maximum width =3.3 km, average width =1.44 km, maximum depth =500 m,average depth =202 m, water storage capacity = 17 km³. It is known that Sarez Lake is very high seismicity zone and is provided by numerous trace of seismodislocation located along the margin of the lake Zones of Lake Sarez are commensurable with size of the foci of large earthquakes and it is situated at the inter action of two large seismo -generating zones Bartang-Pshart and Sares-Zulumart. In connection with we shall consider the possibility of an overflow that might entail the destruction of the weakest part of the dam due to the attacks of powerful flood waves. Such waves can be generated by earthquake shocks and large landslide masses that might be shaken loose to fall into the lake bellow. In both cases the thus generating waves may produce a spillover effect, speed up the process of scouring of the weakest right wing part of the dam, cut a breach in the barrage and trigger a catastrophic mud slide. Now shall bring data of Sarez Lake. Initial dynamical characteristics and parameters are next: 1). The distance from new formation of the mass of oval with volumes 0.9; 0.6; 0.3 km³ up to Usoi oval is equals to 5 km; 2). The top of supposed oval with volumes 0.9km³ has coordinates \( x = 0, h_0 = 3260m; 3 \). Coordinates of Usoi oval are \( x_o, s = 3294m, h_o, s = 5000m \) . The top of the initial top is equals to 314 m for oval with volume 0.9 km³. These data and others initial parameters of the Sarez Lake were taken from experiments works. In order to find out how the waves in Lake Sarez might respond to a sudden fall of large dislodged mass of rock into lake and it is expected to realize its dangerous potential, we shall construct mathematical model of Lake Sarez parameters. Main parameters of Sarez Lake are volume of the landslide (km3), overflow volume (mln.m³), height of the wave (m),the energy of wave the amplitude of the wave, the size of the displaced masses, speed at which various volumes of displaced masses will move, impact of waves of different magnitude on the dam of Lake Sarez. We shall consider Mathematical Model of average on the crows section of flow speed projection-\( u(x,t) \) and height of flow wave-\( h(x,t) \). It is easy that its are solutions of next differential equations (the Continuity and Moving equations):

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = g \sin \psi - g \frac{\partial h}{\partial x} - \left( \frac{x}{\rho h} + \frac{k}{2h^2} \right) \text{sgn} u,
\]

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} = -h \frac{\partial u}{\partial x}, 0 \leq t \leq t_k, x_0 \leq x \leq X_\phi(t),
\]

\[
u(x,0) = u_0(x), h(x,0) = h_0(0), x_0 \leq x \leq X_\phi(0),
\]

\[
u(x_0,0) = 0, h(x_0,0) = 0, 0 \leq t \leq t_k,
\]

\[
\frac{\partial X_\phi}{\partial t} = F(X_\phi(t), t), h(X_\phi, t) = 0, X_\phi(0) = x_0,
\]

where \( \rho \) is the density of flow, \( \psi \) is locus angle of inclination’s path to horizon, \( k \) is a coefficient of hydraulic impedance, \( X = X_\phi(t) \) is a state of flow front in the moment \( t, g \) is acceleration of gravity (accelerations of free incidence), \( \tau \) is
a force friction which is determined in the following way:

\[ \tau = \begin{cases} 
fp & \text{Coulomb law, } \quad (\text{line case}), \\
fp & \text{at } fp \leq \tau^* \text{and} \\
\tau^* & \text{at } fp \geq \tau^*, \text{Grigorian law}, \\
\tau^s \tau^s & \text{our law, } \quad s \geq 0 
\end{cases} \]

Here \( p \) is a pressure, \( f \) is pertaining to Coulomb coefficient \( f = \tau^* / \tau_0 = \tau \) under \( p \to 0 \), \( \tau^* = \lim \tau \) under \( p \to \infty \), \( s = \text{const} \geq 0 \). After integration of these equations under condition \( v \to 0 \) we have:

\[ u(x, t) = u^2(0, t - x/u)e^{-k\int_{0}^{x}u(t+y/u-x/u) + \int_{0}^{x}g\sin\psi(y) - \frac{2\pi}{\rho h(z+t+y/u-x/u)} - g\frac{\partial h}{\partial x}e^{-k\int_{0}^{x}u(t+y/u-x/u)} + \int_{0}^{x}h(x, t) = h_0(x) + \int_{0}^{x}u(X_0(t), y)dy, \quad h(x, t) = h(x, t)u(x, t) = h(x, t - x/u + x_0/u)u(x, t - x/u + x_0/u). \]

As \( \int_{0}^{x}e^{-k\int_{0}^{x}u(t+y/u-x/u)} - k \int_{0}^{x}he^{-k\int_{0}^{x}u(t+y/u-x/u)} \) then we can define functions \( u(x, t), b(x, t), X_0(t) \). Besides.

It is noticed that \( Q(x, t) = h(x, t)u(x, t) \) is defined.

Computer experiments. Under \( t = 10 \) the waves altitude is equal to 306m. Others results are are caries out in the following way:

The waves altitude:

\[
\begin{align*}
\text{The oval volume} & \quad 15\text{sec.} \quad 30\text{sec.} \quad 60\text{sec.} \\
0.9k{\text{m}^3} & \quad 249m \quad 185m \quad 137m \\
0.6k{\text{m}^3} & \quad 139m \quad 97m \quad 71m \\
0.3k{\text{m}^3} & \quad 39m \quad 28m \quad 13m
\end{align*}
\]

The volume of overflow perturbed water:

\[
\begin{align*}
\text{The oval volume} & \quad 15\text{sec.} \quad 30\text{sec.} \quad 60\text{sec.} \\
0.9k{\text{m}^3} & \quad 178k{\text{m}^3} \quad 197k{\text{m}^3} \quad 200k{\text{m}^3} \\
0.6k{\text{m}^3} & \quad 091k{\text{m}^3} \quad 106k{\text{m}^3} \quad 107k{\text{m}^3} \\
0.3k{\text{m}^3} & \quad 0.0k{\text{m}^3} \quad 0.0k{\text{m}^3} \quad 0.007k{\text{m}^3}
\end{align*}
\]

Besides some others computer experiments are also carried out. The equation of (1), the condition of (2) and the representation for \( Q(x, t) \) is correct in the case of \( x \in G \in E^n \). In this case we have next equation:

\[ Q(x_1, x_2, ..., x_n, t) = Q(0, 0, ..., t - \frac{1}{n} \sum_{i=1}^{n} \frac{x_i}{u}) \]

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